

Numerical errors in weight vector computation

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Weight vector computation problem

Minimize array output in all directions but that of the target direction d

Minimization problem (1):



C is signal covariance matrix

$$\text{Solution: } w = \frac{C^{-1}d}{d^H C^{-1}d}$$

C is approximated by

$$A = \sum S_k S_k^H = X^H X \text{ where } X = [S_1, S_2, \dots, S_k]$$

Minimization problem (2):



A is sample covariance matrix

$$\text{Solution: } W = \frac{A^{-1}d}{d^H A^{-1}d}$$

$A = X^H X$. Minimization problem (3):



constrained least squares

Algorithms for (2)

$A = X^H X > 0$, need to compute $A^{-1}d$

Normal Equations (NE)		Semi NE
(I) Cholesky	(II) GE	(III) QR
1) $A = U^H U$ 2) $U^H U = d$ 3) $U v = u$ 4) $w = v/(d^H v)$	1) $A = L^H U$ 2) $L^H U = d$ 3) $U v = u$ 4) $w = v/(d^H v)$	1) $X = QR$ 2) $R^H U = d$ 3) $R v = u$ 4) $w = v/(d^H v)$

Computed A may become indefinite
so use GE instead of Cholesky

Algorithms for (3) – Null Space Method

$$1) Q^H \begin{bmatrix} d^H \\ X \end{bmatrix} H = \begin{bmatrix} 0 & 0 \\ e_1^T & 0 \\ L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix}$$

Generalized
QL decomposition

- 2) $L_{22}w_2 = -L_{21}$ (L_{22} is better conditioned than X)
- 3) $w = H [1 \ w_2^H]^H$
- 4) $\|Xw\| = \|L_{11}\|$ (L_{11} is the norm of the residual)

Sensitivity Analysis - NE

A replaced by $\hat{A} = A + \Delta A$, $\|\Delta A\| < \varepsilon \|A\|$

$$w = \frac{A^{-1}d}{d^H A^{-1}d} \quad \hat{w} = \frac{\hat{A}^{-1}d}{d^H \hat{A}^{-1}d}$$

Then

$$\hat{w} = \frac{\hat{A}^{-1}d}{d^H \hat{A}^{-1}d} = \frac{\hat{A}^{-1}d}{d^H (\hat{A}^{-1})^H \hat{A}^{-1}d} = \frac{\hat{A}^{-1}d}{d^H (\hat{A}^{-1})^H \hat{A}^{-1}d} = \frac{\hat{A}^{-1}d}{d^H (\hat{A}^{-1})^H \hat{A}^{-1}d}$$

Sensitivity Analysis - SNE

X replaced by $\hat{X} = X + \Delta X$, $\|\Delta X\| < \varepsilon \|X\|$

$$w = \frac{(X^H X)^{-1} d}{d^H (X^H X)^{-1} d}$$
$$\hat{w} = \frac{(\hat{X}^H \hat{X})^{-1} d}{d^H (\hat{X}^H \hat{X})^{-1} d}$$

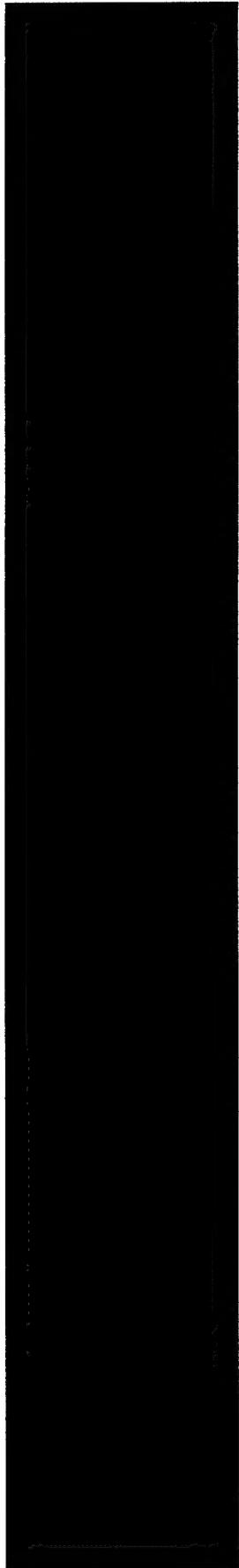
$$\hat{w} = \frac{(\hat{X}^H \hat{X})^{-1} d}{d^H (\hat{X}^H \hat{X})^{-1} d}$$

can be small

Sensitivity Analysis - NS

X replaced by $\hat{X} = X + \Delta X$, $\|\Delta X\| < \varepsilon \|X\|$

$$\min_{\hat{w}^H d = 1} \|\hat{X} \hat{w}\|_2$$



SNE&NS - Small residual case

$$X = W\Sigma V^H, \quad \Sigma = \text{diag}(\sigma_i), \quad V = [v_1, \dots, v_n]$$

If $d = v_n$ then $w = v_n$, $\|Xw\| = \sigma_n$ and



Conclusions

- SNE and NS are equally accurate
 - this is not the case for general LS problems
- If $\|Xw\| = \sigma_n$ then SNE and SN are more accurate than NE
 - in NE use GE instead of Cholesky
- If $\|Xw\| = \sigma_1$ then NE, SNE and SN are equal
 - NE is the least expensive
- cond number determined by submatrices of L
 - cond number can be small even if that of X is large